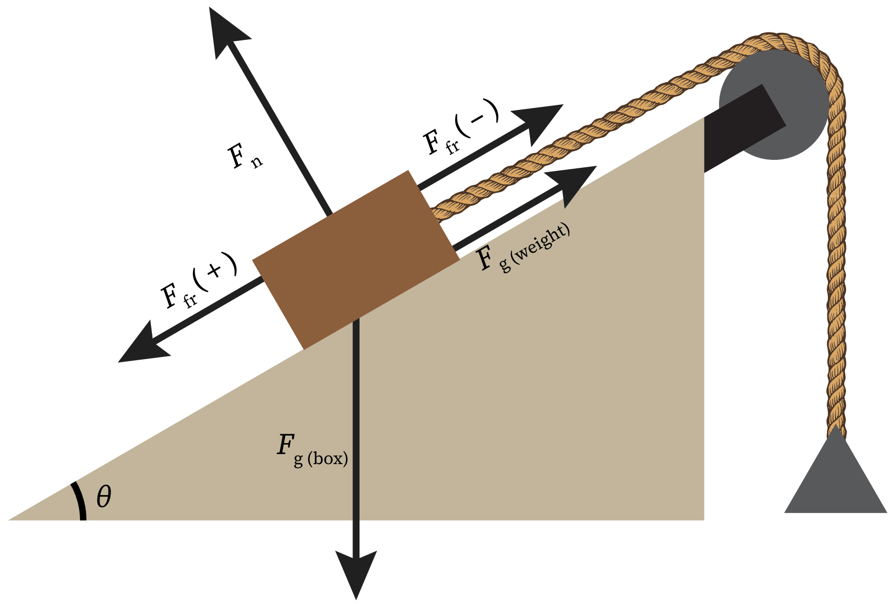


QUESTION

Consider a box on an inclined plane. The box is attached to a weight that is hanging freely via a rope strung over a pulley. Given the coefficient of static friction between the box and the incline, the mass of the box, and the angle of the incline, how heavy must the hanging weight be for the box to attain static equilibrium?

VALUES

The given values are

$$\mu_{static} = 0.2$$
$$m_{box} = 20 \text{ kg}$$
$$\theta = 35°$$

The value we are solving for is

$$m_{weight} = ?$$

SOLUTION

Static equilibrium is when the sum of the forces acting on an object is equal to zero. Since we wish to attain static equilibrium, we may set up our equation this way.

$$\Sigma F = 0$$

Since the only movement that would be happening is along the incline, we are only interested in the sum of the forces in the x direction.

$$\Sigma F_x = 0$$

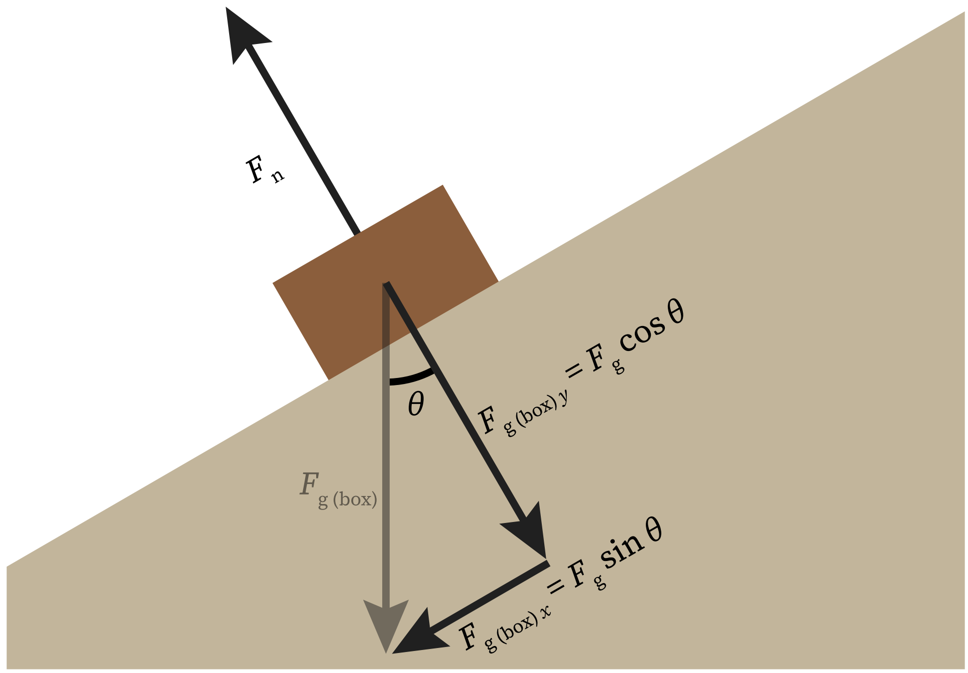
The forces acting on the box are the force of gravity ($F_{\text{g (box)}}$), which can be split up in its x and y directions ($F_{\text{g (box)}x}$ and $F_{\text{g (box)}y}$), the force of friction acting on the box (F_{fr}), the normal force (F_{n}), and the force of gravity acting on the weight, which is pulling on the box through the pulley ($F_{\text{g (weight)}}$). Let's define moving "down" the incline as the positive x direction, and moving "up" the incline as the negative x direction. Since we are only interested in the x direction, the sum of the forces acting on the box in the x direction can be substituted and rearranged.

$F_{\text{g (box)}x}$ is acting to pull the box down the incline, so it is acting in the positive x direction. $F_{\text{g (weight)}}$ is acting to pull the box up the incline, so it is acting in the negative x direction. F_{fr} is acting in such a way as to resist the movement of the box, whatever direction along the incline that may be. If the weight is too light, the box will begin sliding down the incline. If the weight is too heavy, the box will begin sliding up the incline. So, depending on the situation, F_{fr} will be acting in either the negative x direction or the positive x direction. Let's substitute our forces and rearrange the equation.

$$F_{\text{g (box)}x} \pm F_{\text{fr}} - F_{\text{g (weight)}} = 0$$
$$F_{\text{g (box)}x} \pm F_{\text{fr}} = F_{\text{g (weight)}}$$

Now let's substitute our known values.

We know $F_{\text{g (box)}}$ is equal to $m_{\text{box}}g$. Since we know the angle of the incline, we can solve for $F_{\text{g (box)}x}$ using the sine of the angle of the incline to get $m_{\text{box}}g\sin\theta$ (see the diagram on the next page).



We know F_{fr} is equal to μF_n. In this scenario, we know that F_n is equal in magnitude but opposite in direction to $F_{g\,(box)\,y}$, giving us $\mu F_{g\,(box)\,y}$. Like before, we can solve for $F_{g\,(box)\,y}$ using the cosine of the angle of the incline to get $m_{box} g \cos\theta$, making F_{fr} equal to $\mu m_{box} g \cos\theta$ (see the diagram above).

$F_{g\,(weight)}$ is equal to $m_{weight} g$.

Let's substitute these known values into our equation.

$$m_{box} g \sin\theta \pm \mu m_{box} g \cos\theta = m_{weight} g$$

The gravitational acceleration constant g is found in every term on both sides of the equals sign, so these terms reduce, simplifying our equation.

$$m_{box} \sin\theta \pm \mu m_{box} \cos\theta = m_{weight}$$

We have now isolated m_{weight} which is the value we are solving for, but we can further simplify the equation by rearranging and factoring out m_{box} to make calculating easier.

$$m_{weight} = m_{box} (\sin\theta \pm \mu \cos\theta)$$

By first solving the (+) part of the equation (where friction is acting against the box sliding *up* the incline), we get

$$m_{\text{weight}} = 14.75 \text{ kg}.$$

Any weight heavier than 14.75 kg will cause the box to slide up the incline.

By solving the (–) part of the equation (where friction is acting against the box sliding *down* the incline), we get

$$m_{\text{weight}} = 8.20 \text{ kg}.$$

Any weight lighter than 8.20 kg will cause the box to slide down the incline.

ANSWER

To keep the box in static equilibrium, the mass of the weight must be anywhere between 14.75 kg and 8.20 kg.